Nucleon and Δ resonances in $K\Sigma(1385)$ photoproduction from nucleons

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The reaction mechanisms for $K\Sigma(1385)$ photoproduction from the reaction $\gamma p \to K^+\Sigma^0(1385)$ in the resonance energy region are investigated in a hadronic model. Both contributions from N and Δ resonances of masses around 2 GeV as given in the Review of Particle Data Group and by the quark model predictions are included. The Lagrangians for describing the decays of these resonances into $K\Sigma(1385)$ are constructed with the coupling constants determined from the decay amplitudes predicted by a quark model. Comparing the resulting total cross section for the reaction $\gamma p \to K^+\Sigma^0(1385)$ with the preliminary data from the Thomas Jefferson National Accelerator Facility, we find that the most important contributions are from the two-star rated resonances $\Delta(2000)F_{35}$, $\Delta(1940)D_{33}$, and $N(2080)D_{13}$, as well as the missing resonance $N\frac{3}{2}^-(2095)$ predicted in the quark model. Predictions on the differential cross section and photon asymmetry in this reaction are also given.

I. INTRODUCTION

Strangeness production from photon-nucleon reactions has been extensively studied in recent experiments at electron/photon accelerator facilities [1–4]. Among the motivations for such studies are to obtain a deeper understanding of the baryon resonances and to search for the so-called missing resonances, whose existence is predicted by quark models but has not been experimentally confirmed. Most of the data from these experiments are for reactions of kaon photoproduction which are accompanied by the ground state of Λ or Σ hyperon, i.e., $\Lambda(1116)$ or $\Sigma(1193)$. Recently, there have been reports on experimental studies of other strangeness production processes that include $K^*\Lambda$, $K^*\Sigma$, and $K\Sigma(1385)$ photoproduction [5–7]. Although the reported cross sections for these reactions are smaller than those for $K\Lambda(1116)$ and $K\Sigma(1193)$ photoproduction, the suppression factor is not large. In fact, the magnitude of the cross sections for these reactions in the resonance region, corresponding to total center-of-mass energies around 2 GeV, is as large as one half of the $K\Lambda(1116)$ and $K\Sigma(1193)$ photoproduction cross sections. This indicates that these reaction channels cannot be neglected in a full coupled channel calculation for extracting the properties of these baryon resonances [5]. In addition, these reactions have their own interesting physics regarding the structure of hadrons. For example, photoproduction of $K^*\Lambda$ and $K^*\Sigma$ can be used to obtain information on the properties of strange scalar κ meson [8, 9].

Regarding the missing resonance problem, photoproduction of $K\Sigma(1385)$ provides a useful tool for testing baryon models in the literature. According to the quark model of Ref. [10], most nucleon and Δ resonances have small couplings to the $K\Sigma(1385)$ channel. Some resonances, mostly missing or not-well-established ones, are, however, predicted to have large partial decay widths into this channel. For example, the missing resonance $N\frac{3}{2}^-(2095)$ was predicted to have a decay width of $\Gamma\big(N\frac{3}{2}^-(2095) \to K\Sigma(1385)\big) \approx 60$ MeV. Therefore, photoproduction of $K\Sigma(1385)$ could be an ideal reaction in which one can search for such resonances.

Experimental studies of $K\Sigma(1385)$ photoproduction are very rare, and only limited experimental data on the total cross section for $\gamma p \to K^+\Sigma^0(1385)$ with large error bars have been reported [11–13]. The CLAS Collaboration at the Thomas Jefferson National Accelerator Facility recently measured the cross section of this reaction at 23 different photon energies covering from the threshold up to 3.8 GeV [5]. More accurate data for the total and differential cross sections are expected to be reported soon [14]. We will use the preliminary data for the total cross section of this reaction reported in Ref. [5] for our study.

Theoretical investigation of $K\Sigma(1385)$ photoproduction is also very scarce. To our knowledge, only a few theoretical studies on this reaction were reported quite recently. In Ref. [15], contributions from the single and double K-meson pole terms to the differential cross section of this reaction were compared, while the role of $\Delta(1700)$ resonance near the threshold region was addressed in Ref. [16]. In this paper, we present a model for $K\Sigma(1385)$ photoproduction from the reaction $\gamma p \to K^+\Sigma^0(1385)$, based on an effective Lagrangian approach. In addition to the t-channel K and K^* meson exchanges, we consider the s- and u-channel diagrams as well as the contact term, which are required by crossing symmetry and the gauge invariance condition. We also investigate the role of resonances in this reaction. For this purpose

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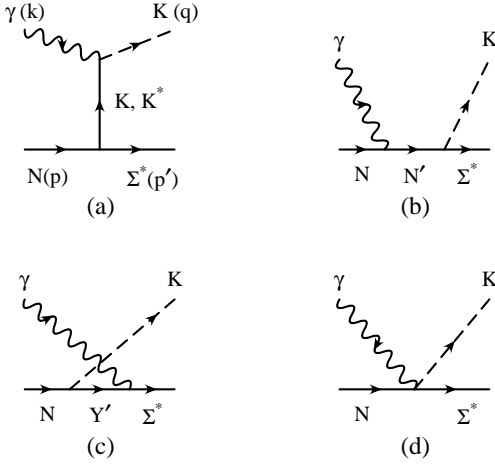‡Electronic address: nakayama@uga.edu

FIG. 1: Feynman diagrams for $\gamma p \to K^+ \Sigma^{*0}$. N' stands for the nucleon, Δ, and their resonances, and Y' the Λ, Σ, and their resonances. For details of the contact term [diagram (d)] used in the present work, see Eq. (15).

we construct the Lagrangians involving the decay of resonances into $K\Sigma(1385)$. The coupling constants in these Lagrangians are determined by decomposing the decay amplitudes according to the relative orbital angular momentum of the final $K\Sigma(1385)$ state and then comparing them with those known empirically or calculated from hadronic models.

This paper is organized as follows. In Sec. II, we discuss the effective Lagrangians employed in the present work. This includes the general form for the $K^*N\Sigma^*$ interactions (or $\rho N\Delta$ interactions), which has been overlooked in the literature. Numerical results on the total and differential cross sections as well as the photon asymmetry are presented and discussed in Sec. III, which is followed by a summary and discussion in Sec. IV. The propagators of spin-3/2 and -5/2 baryons are given in Appendix A together with the isospin structure of the interaction Lagrangians. Given in Appendix B are details on the decay amplitudes of baryon resonances into $K\Sigma(1385)$ and $N\gamma$, which are used to relate the coupling constants in the interaction Lagrangians to the predicted decay amplitudes from hadronic models.

II. THE MODEL

A. Effective Lagrangians

Particle production in photon-nucleon interactions has been extensively studied in hadronic models based on effective Lagrangians. This includes the production of various mesons [17], charmed hadrons [18], Ξ baryons [19], and exotic baryons [20, 21]. In this paper, we use this approach to study the reaction $\gamma p \to K^+ \Sigma^0(1385)$. The production mechanisms for this reaction are shown in Fig. 1. Figure 1(a) includes the t-channel K and

K^* meson exchange diagrams. The s-channel diagrams shown in Fig. 1(b) contain contributions from non-strange baryons, i.e., nucleon, Δ and their resonances. In the present work, we consider resonances of masses around 2 GeV as the purpose of this work is to investigate the role of such resonances in this reaction. Resonances below the $K\Sigma(1385)$ threshold are not considered since there is no information on their couplings to the $K\Sigma(1385)$ channel. The u-channel diagrams shown in Fig. 1(c) contain hyperons and their resonances. Although all Λ and Σ resonances can contribute to the reaction through this diagram, only $\Lambda(1116)$ and $\Sigma(1385)$ are considered in the present study as there is no information on the photo-transitions between $\Sigma(1385)$ and hyperon resonances with masses around 2 GeV. Figure 1(d) is the contact diagram required by gauge invariance.

The production amplitudes from the diagrams for t-channel K exchange, s-channel nucleon term, u-channel $\Sigma(1385)$ term, and the contact term can be calculated from following effective Lagrangians:

$$\mathcal{L}_{\gamma KK} = ieA_\mu \left(K^- \partial^\mu K^+ - \partial^\mu K^- K^+ \right),$$

$$\mathcal{L}_{KN\Sigma^*} = \frac{f_{KN\Sigma^*}}{M_K} \partial_\mu \overline{K\Sigma}^{*\mu} \cdot \boldsymbol{\tau} N + \text{ H.c.},$$

$$\mathcal{L}_{\gamma NN} = -e\bar{N} \left(\gamma^\mu A_\mu \frac{1+\tau_3}{2} - \frac{\kappa_N}{2M_N} \sigma^{\mu\nu} \partial_\nu A_\mu \right) N,$$

$$\mathcal{L}_{\gamma KN\Sigma^*} = -ie\frac{f_{KN\Sigma^*}}{M_K} A^\mu K^- \left(\bar{\Sigma}_\mu^{*0} p + \sqrt{2}\bar{\Sigma}_\mu^{*+} n \right) + \text{ H.c.},$$

$$\mathcal{L}_{\gamma\Sigma^*\Sigma^*} = e\overline{\Sigma}_\mu^* A_\alpha \Gamma_{\gamma\Sigma^*}^{\alpha,\mu\nu} \Sigma_\nu^*, \tag{1}$$

where M_K is the kaon mass, A_μ is the photon field, and Σ_μ^* is the Rarita-Schwinger field for the $\Sigma(1385)$ of spin-3/2.[1] The isodoublets are defined as

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \qquad \overline{K} = \left(K^-, \bar{K}^0 \right), \qquad N = \begin{pmatrix} p \\ n \end{pmatrix}. \tag{2}$$

The electromagnetic interaction of the Σ^* field contains

$$A_\alpha \Gamma_{\gamma\Sigma^*}^{\alpha,\mu\nu} = \left\{ g^{\mu\nu}\gamma^\alpha - \frac{1}{2} \left(\gamma^\mu\gamma^\nu\gamma^\alpha + \gamma^\alpha\gamma^\mu\gamma^\nu \right) \right\} A_\alpha T_3$$
$$- \frac{\kappa_{\Sigma^*}}{2M_N} \sigma^{\alpha\beta} \partial_\beta A_\alpha g^{\mu\nu}, \tag{3}$$

where M_N is the nucleon mass and $T_3 = \text{diag}(1, 0, -1)$.

For the $KN\Sigma^*$ coupling, it can be related to the $\pi N\Delta$ coupling by the SU(3) flavor symmetry relation,

$$\frac{f_{\pi N\Delta}}{M_\pi} = -\sqrt{6}\frac{f_{KN\Sigma^*}}{M_K}, \tag{4}$$

where M_π is the pion mass. Estimating the $\pi N\Delta$ coupling as $f_{\pi N\Delta} = 2.23$ from the Delta resonance decay

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[1] In this work, we do not consider the off-shell properties of the Rarita-Schwinger field.

width $\Gamma(\Delta \to N\pi) = 120$ MeV, we obtain from the above equation $f_{KN\Sigma^*} = -3.22$. The electromagnetic interactions of baryons contain the baryon anomalous magnetic moments. We use the empirical value $\kappa_p = 1.793$ for the proton. Since the magnetic moment of $\Sigma^0(1385)$ is unknown, its value is taken from the quark model prediction given in Ref. [22], i.e., $\kappa_{\Sigma^{*0}} = 0.36$.

For the t-channel K^* exchange, we use the Lagrangian,

$$\mathcal{L}_{\gamma KK^*} = g_{K^*K\gamma}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu \partial_\alpha K^{*-}_\beta \bar{K}^+ + \text{ H.c.}, \quad (5)$$

for the γKK^* interaction with $g_{K^*K\gamma} = 0.254$ GeV^{-1}, which is determined from the empirical value of the K^* decay width $\Gamma(K^{*\pm} \to K^\pm\gamma) \approx 50$ keV. We note that for neutral K^*, $g_{K^*K\gamma} = -0.388$ GeV^{-1} as $\Gamma(K^{*0} \to K^0\gamma) \approx 116$ keV, and the signs of these coupling constants are fixed by the quark model.

For the interactions of a vector meson with spin-1/2 and spin-3/2 baryons, i.e., for the vertex of $\frac{3}{2} \to 1 + \frac{1}{2}$, there are in general three independent interaction terms from consideration of angular momentum and parity conservation. The most general form of the $K^*N\Sigma^*$ interaction Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_{K^*N\Sigma^*} &= \frac{ig_1}{2M_N}\overline{K}^{*\mu\nu}\overline{\Sigma}^*_\mu \cdot \boldsymbol{\tau}\gamma_\nu\gamma_5 N \\ &+ \frac{g_2}{(2M_N)^2}\overline{K}^{*\mu\nu}\overline{\Sigma}^*_\mu \cdot \boldsymbol{\tau}\gamma_5\partial_\nu N \\ &- \frac{g_3}{(2M_N)^2}\partial_\nu\overline{K}^{*\mu\nu}\overline{\Sigma}^*_\mu \cdot \boldsymbol{\tau}\gamma_5 N + \text{ H.c.},(6)\end{aligned}$$

where $K^*_{\mu\nu} = \partial_\mu K^*_\nu - \partial_\nu K^*_\mu$ and K^* is an isodoublet as K in Eq. (2). To determine the coupling constants $g_{1,2,3}$, we again make use of the SU(3) relations to relate them to the $\rho N\Delta$ coupling. For the coupling constant g_1, the SU(3) relation

$$\frac{g_1^{\rho N\Delta}}{M_\rho} = -\sqrt{6}\frac{g_1^{K^*N\Sigma^*}}{2M_N} \quad (7)$$

leads to $g_1 = -5.48$ for the $K^*N\Sigma^*$ coupling if the empirically determined value $g_1^{\rho N\Delta} = 5.5$ [23, 24] is used. Since the other two couplings, g_2 and g_3, in the $\rho N\Delta$ interactions have never been seriously considered in previous studies, corresponding couplings for the $K^*N\Sigma^*$ interactions thus cannot be determined. In the present study, we treat g_2 and g_3 in the $K^*N\Sigma^*$ interactions as free parameters and vary their values to find their role in $K\Sigma(1385)$ photoproduction.

The u-channel diagrams shown in Fig. 1(c) contain intermediate hyperon Y'. Because of the lack of information on the radiative decays of hyperon resonances to $\Sigma(1385)$, we consider only the contribution from the ground state hyperons. The effective Lagrangians for these diagrams are

$$\begin{aligned}\mathcal{L}_{\Sigma^*Y\gamma} &= -\frac{ief_1}{2M_Y}\overline{Y}\gamma_\nu\gamma_5 F^{\mu\nu}\Sigma^*_\mu \\ &\quad - \frac{ef_2}{(2M_Y)^2}\partial_\nu\overline{Y}\gamma_5 F^{\mu\nu}\Sigma^*_\mu + \text{ H.c.}, \\ \mathcal{L}_{KNY} &= \frac{g_{KNY}}{M_N+M_Y}\overline{N}\gamma^\mu\gamma_5 Y\partial_\mu K + \text{ H.c.,} \quad (8)\end{aligned}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and Y stands for a hyperon with spin-1/2. For the intermediate $\Lambda(1116)$ state, we use the radiative decay width, $\Gamma(\Sigma(1385) \to \Lambda\gamma) = 479 \pm 120$ keV, as recently measured by CLAS Collaboration. To estimate the relative strength of the two coupling constants, we make use of the quark model result of Ref. [26] for the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ (see Appendix B), which gives the ratio $A_{3/2}/A_{1/2} \approx 1.82$.[2] Combining the two information, we obtain

$$f_1 = 4.52, \qquad f_2 = 5.63. \quad (9)$$

The coupling constant $g_{KN\Lambda}$ can be determined by flavor SU(3) symmetry relations, which give $g_{KN\Lambda} = -13.24$. For the intermediate $\Sigma(1193)$ hyperon, there is no experimental data for $\Gamma(\Sigma(1385) \to \Sigma(1193)\gamma)$. However, most hadron model calculations show that the decay width of $\Sigma(1385) \to \Sigma(1193)\gamma$ is less than 10% of that of $\Sigma(1385) \to \Lambda(1116)\gamma$ [26, 27]. With $g_{KN\Sigma} = 3.58$ from the flavor SU(3) symmetry, we have estimated that the contribution from the u-channel $\Sigma(1193)$ is only at the level of 0.5% of the u-channel $\Lambda(1116)$ contribution. In the present work, therefore, we will consider the u-channel diagram with the $\Lambda(1116)$ hyperon only.

As we have mentioned above, the motivation for the study of $K\Sigma(1385)$ photoproduction is to identify the contributions from baryon resonances. For this purpose, we consider the contributions from both nucleon and Δ resonances in the present work, which requires the interaction Lagrangians for photoexcitation of a resonance from a nucleon as well as for its decay into $K\Sigma(1385)$ channel. For the former, we use

$$\begin{aligned}\mathcal{L}_{RN\gamma}(\tfrac{1}{2}^\pm) &= \frac{ef_1}{2M_N}\bar{N}\Gamma^{(\mp)}\sigma_{\mu\nu}\partial^\nu A^\mu R + \text{H.c.}, \\ \mathcal{L}_{RN\gamma}(\tfrac{3}{2}^\pm) &= -\frac{ief_1}{2M_N}\overline{N}\Gamma^{(\pm)}_\nu F^{\mu\nu}R_\mu \\ &\quad - \frac{ef_2}{(2M_N)^2}\partial_\nu\bar{N}\Gamma^{(\pm)}F^{\mu\nu}R_\mu + \text{H.c.}, \\ \mathcal{L}_{RN\gamma}(\tfrac{5}{2}^\pm) &= \frac{ef_1}{(2M_N)^2}\bar{N}\Gamma^{(\mp)}_\nu\partial^\alpha F^{\mu\nu}R_{\mu\alpha} \\ &\quad - \frac{ief_2}{(2M_N)^3}\partial_\nu\bar{N}\Gamma^{(\mp)}\partial^\alpha F^{\mu\nu}R_{\mu\alpha} + \text{H.c.,}\end{aligned}$$
$$(10)$$

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[2] We note that most theoretical predictions [26, 27] underestimate this decay width.

where R, R_μ, and $R_{\mu\nu}$ are the fields for the spin-1/2, 3/2, and 5/2 resonances, respectively, with

$$\Gamma_\mu^{(\pm)} = \begin{pmatrix} \gamma_\mu\gamma_5 \\ \gamma_\mu \end{pmatrix}, \qquad \Gamma^{(\pm)} = \begin{pmatrix} \gamma_5 \\ \mathbf{1} \end{pmatrix}. \qquad (11)$$

It should be noted that the coupling constant f_i has isospin dependence if the resonance R has isospin 1/2, while it is isospin blind if the isospin of R is 3/2. Since we only consider in the present work photoexcitation of resonances from the proton, the isospin quantum number is fixed in the process.

For the decay of a resonance with spin j into $K\Sigma(1385)$, the number of possible interaction terms in the Lagrangian is restricted by the angular momentum and parity conservation. The interaction Lagrangian has one term for a $j = \frac{1}{2}$ resonance but has two terms for a resonance with $j \geq \frac{3}{2}$. Explicitly, the effective Lagrangians for $RK\Sigma^*$ interactions can be written as

$$
\begin{aligned}
\mathcal{L}_{RK\Sigma^*}(\tfrac{1}{2}^\pm) &= \frac{h_1}{M_K}\partial_\mu K\bar{\Sigma}^{*\mu}\Gamma^{(\mp)}R + \text{H.c.,} \\
\mathcal{L}_{RK\Sigma^*}(\tfrac{3}{2}^\pm) &= \frac{h_1}{M_K}\partial^\alpha K\bar{\Sigma}^{*\mu}\Gamma_\alpha^{(\pm)}R_\mu \\
&\quad + \frac{ih_2}{M_K^2}\partial^\mu\partial^\alpha K\bar{\Sigma}_\alpha^*\Gamma^{(\pm)}R_\mu + \text{H.c.,} \\
\mathcal{L}_{RK\Sigma^*}(\tfrac{5}{2}^\pm) &= \frac{ih_1}{M_K^2}\partial^\mu\partial^\beta K\bar{\Sigma}^{*\alpha}\Gamma_\mu^{(\mp)}R_{\alpha\beta} \\
&\quad - \frac{h_2}{M_K^3}\partial^\mu\partial^\alpha\partial^\beta K\bar{\Sigma}_\mu^*\Gamma^{(\mp)}R_{\alpha\beta} + \text{H.c..}
\end{aligned}
$$
$$(12)$$

In evaluating the Feynman diagrams (b) and (c) in Fig. 1 for the reaction $\gamma p \to K^+\Sigma^0(1385)$, we need also the propagators of baryon resonances. They are given explicitly in Appendix A for baryon resonances of spins up to 5/2, together with the isospin structure of their interaction Lagrangians. For the coupling constants $f_{1,2}$ in Eq. (10) and $h_{1,2}$ in Eq. (12), they can be related to the photon helicity amplitudes of $R \to N\gamma$ and the decay amplitudes of $R \to K\Sigma(1385)$, respectively. These relations are given explicitly in Appendix B, and they allow us to determine the coupling constants once these amplitudes are known either empirically or from models for hadrons.

B. Form factors

In evaluating the production amplitudes of $\gamma p \to K^+\Sigma^0(1385)$, we need to dress the interaction vertices with form factors. We use the monopole type form factor for the t-channel K meson exchange diagram, i.e.,

$$F_M(q_{\text{ex}}^2, M_{\text{ex}}) = \frac{\Lambda_M^2 - M_{\text{ex}}^2}{\Lambda_M^2 - q_{\text{ex}}^2}. \qquad (13)$$

For s- and u-channel diagrams and t-channel K^* exchange, we adopt the form factor

$$F_B(q_{\text{ex}}^2, M_{\text{ex}}) = \left(\frac{n\Lambda_B^4}{n\Lambda_B^4 + (q_{\text{ex}}^2 - M_{\text{ex}}^2)^2}\right)^n, \qquad (14)$$

which goes to a Gaussian form as $n \to \infty$. In Eqs. (13) and (14), q_{ex} is the four-momentum of the exchanged particle of mass M_{ex}. The cutoff parameters Λ_M and Λ_B as well as n will be adjusted to fit the experimental data.

C. Generalized contact current

Employing different form factors to interaction vertices breaks gauge invariance. Following the prescription of Ref. [28], we restore gauge invariance by introducing the following generalized contact term to the amplitude for the reaction $\gamma p \to K^+\Sigma^0(1385)$:

$$M_c^{\mu\nu} = \Gamma_{\gamma KN\Sigma^*}^{\mu\nu}f_t + ie\Gamma_{KN\Sigma^*}^\nu(q)C^\mu. \qquad (15)$$

In the above equation,

$$\Gamma_{\gamma KN\Sigma^*}^{\mu\nu} = ie\frac{f_{KN\Sigma^*}}{M_K}g^{\mu\nu} \qquad (16)$$

and

$$\Gamma_{KN\Sigma^*}^\nu(q) = -\frac{f_{KN\Sigma^*}}{M_K}q^\nu, \qquad (17)$$

are vertex functions obtained, respectively, from $\mathcal{L}_{\gamma KN\Sigma^*}$ and $\mathcal{L}_{KN\Sigma^*}$ in Eq. (1) with q being the momentum of the outgoing K meson; C^μ is defined as

$$C^\mu = -(2q-k)^\mu\frac{f_t-1}{t-M_K^2}f_s - (2p+k)^\mu\frac{f_s-1}{s-M_N^2}f_t, \qquad (18)$$

with the momenta defined in Fig. 1, and the form factors in t-channel [Eq. (13)] and s-channel [Eq. (14)] diagrams are denoted by f_t and f_s, respectively. Note that the first term on the right-hand-side of Eq. (15) is the usual Kroll-Ruderman contact current multiplied by the t-channel form factor. The last term is an additional contact current required to restore gauge invariance of the total amplitude in the presence of form factors at the hadronic vertices. We note that the contact current as specified above also satisfies the crossing symmetry. For details on the restoration of gauge invariance and a more general form of C^μ, we refer the readers to Ref. [28].

D. N and Δ resonances

For resonances in the s-channel diagrams, we include those with spin $j \leq 5/2$. Neglecting resonances with higher spins is justified as they have been shown in Ref. [10] to couple weakly to the $K\Sigma(1385)$ channel.

Resonance	PDG [29]	Amplitudes of $R \to K\Sigma(1385)$ †		h_1	h_2	Amplitudes of $R \to N\gamma$ ††		f_1	f_2
		$G(\ell_1)$	$G(\ell_2)$			$A^p_{1/2}$	$A^p_{3/2}$		
$N\frac{1}{2}^-(1945)$	$S^*_{11}(2090)$	$G(2) = +1.7$	—	-9.8	—	$+12$	—	-0.055	—
$N\frac{3}{2}^-(1960)$	$D^{**}_{13}(2080)$	$G(0) = +1.3$	$G(2) = +1.4$	0.24	-0.54	$+36$	-43	-1.25	1.21
$N\frac{5}{2}^-(2095)$	$D^{**}_{15}(2200)$	$G(2) = -2.0$	$G(4) = 0.0$	0.29	-0.08	-9	-14	0.37	-0.57
$\Delta\frac{3}{2}^-(2080)$	$D^*_{33}(1940)$	$G(0) = -4.1$	$G(3) = -0.5$	-0.68	1.00	-20	-6	0.39	-0.57
$\Delta\frac{5}{2}^+(1990)$	$F^{**}_{35}(2000)$	$G(1) = +4.0$	$G(3) = -0.1$	-0.87	0.11	-10	-28	-0.68	-0.062

†: in $\sqrt{\text{GeV}}$

†† in $10^{-3}/\sqrt{\text{GeV}}$

TABLE I: Resonances listed in the review of PDG [29] and their decay amplitudes of $R \to K\Sigma(1385)$ and of $R \to N\gamma$ predicted in Refs. [10, 30]. The coupling constants are calculated using the resonance masses of PDG.

We classify the resonances into two groups: (A) resonances listed in the review of Particle Data Group (PDG) [29] and (B) missing resonances. Since the decay widths of the resonances listed in PDG into $K\Sigma(1385)$ have not been empirically determined, we have to rely on theoretical models, such as the quark model of Refs. [10, 30], to determine their coupling constants to $K\Sigma(1385)$. Instead of all possible resonances, we consider only a few of them which are predicted to have large couplings to $N\gamma$ and to $K\Sigma(1385)$. The resonances of group (A) and the predictions of the quark model given in Refs. [10, 30] on their decay amplitudes are given in Table I. These resonances are referred to as PDG resonances and include $N\frac{1}{2}^-(1945)$, $N\frac{3}{2}^-(1960)$, $N\frac{5}{2}^-(2095)$, $\Delta\frac{3}{2}^-(2080)$, and $\Delta\frac{5}{2}^+(1990)$, which are identified as $N(2090)S^*_{11}$, $N(2080)D^{**}_{13}$, $N(2200)D^{**}_{15}$, $\Delta(1940)D^*_{33}$, and $\Delta(2000)F^{**}_{35}$, respectively, by the authors of Ref. [10]. Although listed in the review of PDG, these resonances are rated as either one-star or two-star resonances, which means that the evidence of their existence is poor or only fair [29] and that further works are required to verify their existence and to know their properties. Accordingly, their total decay widths and branching ratios are not known. In the present work, we assume the same total decay width of $\Gamma_R = 300$ MeV for these resonances.

The missing resonances which are predicted to have large couplings to the $K\Sigma(1385)$ channel are listed in Table II. These resonances include $N\frac{3}{2}^-(2095)$, $N\frac{5}{2}^+(1980)$, and $\Delta\frac{3}{2}^-(2145)$. Among them the resonance $N\frac{3}{2}^-(2095)$ is particularly interesting since it is predicted to have a very large decay width into the $K\Sigma(1385)$ channel, $\Gamma\big(N\frac{3}{2}^-(2095) \to K\Sigma(1385)\big) \simeq$ 60 MeV [10]. Photoproduction of $K\Sigma(1385)$ thus offers an opportunity for finding these resonances. In Ref. [10], the missing resonances $N\frac{1}{2}^-(2070)$ and $N\frac{5}{2}^-(2260)$ are also predicted to have large couplings to $K\Sigma(1385)$, but no prediction for their photoexcitation amplitudes have been made within the same model. We thus leave the investigation on the role of these resonances to a future study. We also assume $\Gamma_R = 300$ MeV for these resonances.
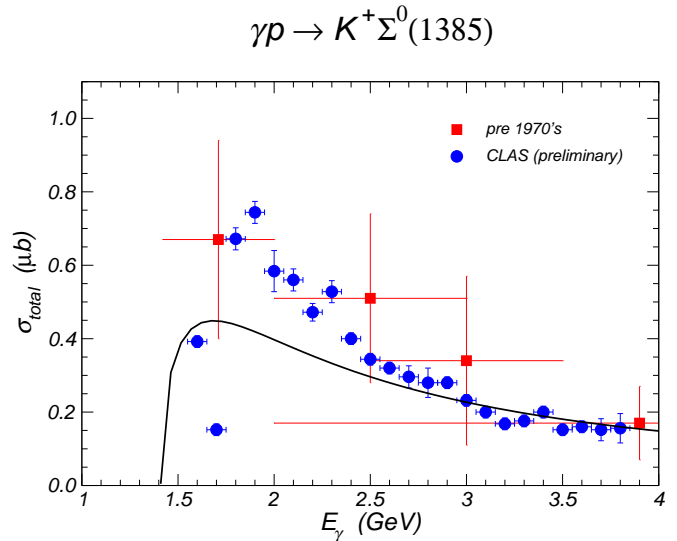
$$\gamma p \to K^+\Sigma^0(1385)$$

FIG. 2: (Color online) Total cross sections for $\gamma p \to K^+\Sigma^0(1385)$ without resonance contributions. The pre-1970s data are from Refs. [11–13] and the preliminary data of CLAS Collaboration are from Ref. [5].

III. RESULTS

A. Total cross section

With the effective Lagrangians and form factors constructed above, we first compute the total cross section for $\gamma p \to K^+\Sigma^0(1385)$ without the resonance contributions. For the form factors in the s- and u-channel diagrams, we take $\Lambda_B = 1.0$ GeV with $n = 1$. For the t-channel K exchange, we use $\Lambda_M = 0.83$ GeV to reproduce the total cross section data at $E_\gamma \geq 2.5$ GeV. Following Ref. [31], we avoid the use of F_M for vector meson exchanges and the t-channel K^* exchange is calculated by using the form factor F_B with $\Lambda_B = 1.2$ GeV and $n = 1$. With the $K^*N\Sigma^*$ coupling constants determined before, namely, $g_1 = -5.48$ and $g_2 = g_3 = 0$, we find that the contribution from the K^* exchange is negligible in the considered energy region. Even at higher

Resonance	Amplitudes of $R \to K\Sigma(1385)$ †		h_1	h_2	Amplitudes of $R \to N\gamma$ ††		f_1	f_2
	$G(\ell_1)$	$G(\ell_2)$			$A^p_{1/2}$	$A^p_{3/2}$		
$N\frac{3}{2}^-(2095)$	$G(0)=+7.7$	$G(2)=-0.8$	0.99	0.27	-9	-14	0.49	-0.83
$N\frac{5}{2}^+(1980)$	$G(1)=-3.6$	$G(3)=-0.1$	0.59	0.24	-11	-6	0.019	-0.13
$\Delta\frac{3}{2}^-(2145)$	$G(0)=+5.2$	$G(2)=-1.9$	0.25	0.46	0	$+10$	0.11	-0.059

†: in $\sqrt{\mathrm{GeV}}$
†† in $10^{-3}/\sqrt{\mathrm{GeV}}$

TABLE II: Missing resonances and their decay amplitudes predicted in Refs. [10, 30].

energies, $E_\gamma = 3 \sim 4$ GeV, the K^* exchange contribution is only at the level of a few percent of those from other production mechanisms. We have also tested the role of the K^* exchange by allowing non-vanishing values for g_2 and g_3. We again find that the K^* exchange is suppressed compared with other production mechanisms unless g_2 and/or g_3 is as large as ~ 100. Although there is no constraint at present on the values of g_2 and g_3, we regard such a large value unrealistic. This leads us to conclude that the role of K^* exchange in this reaction is negligibly small. However, since the K^* trajectory has a larger intercept than the K trajectory, the role of the K^* exchange would have a chance to be revealed at very high energies. It is thus of interest to measure the cross sections at much higher energies, and this would help constrain the values of the coupling constants g_2 and g_3.

Our result on the total cross section is shown in Fig. 2 and is compared with the pre-1970's data [11–13] and the preliminary CLAS data reported in Ref. [5].[3] Comparison with the preliminary CLAS data for the total cross section of $\gamma p \to K\Sigma^0(1385)$ shows that this model can explain the general energy dependence of the total cross section but not the enhanced cross section at $E_\gamma = 1.7 \sim 1.9$ GeV. Although varying the cutoff parameters of employed form factors can change the magnitude of the cross section, the peak arising from the threshold effect cannot reproduce the observed peak in the data. This implies that resonances play an important role in the production mechanism.

Including the s-channel nucleon and Δ resonances listed in Tables I and II in the reaction $\gamma p \to K^+\Sigma^0(1385)$, we have recalculated its cross section. In this calculation, the parameters of the non-resonant terms are fixed as before, while the resonance terms are obtained by using the form factor F_B in the form of the Gaussian function that is obtained by taking $n \to \infty$ and the cutoff $\Lambda_B = 1.0$ GeV, as motivated by the Gaussian radial wave functions in the quark model. The resulting total cross section for the reaction $\gamma p \to K^+\Sigma^0(1385)$ is shown in Fig. 3. As shown by the dashed line, the
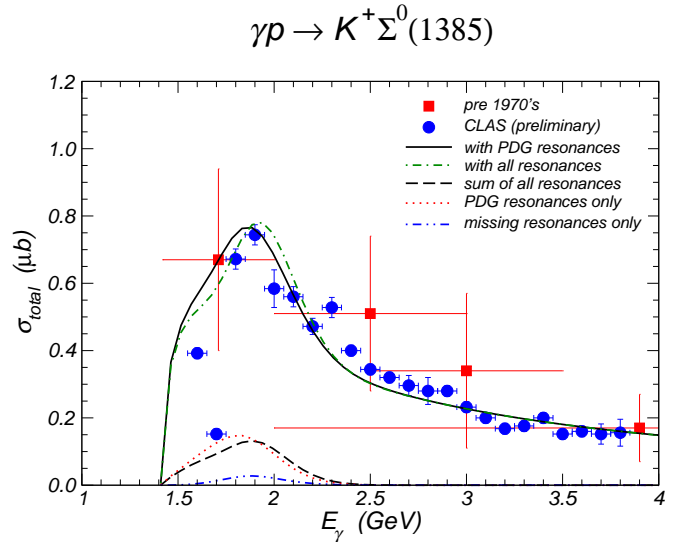
FIG. 3: (Color online) Total cross sections for the $\gamma p \to K^+\Sigma^0(1385)$ reaction with the resonances listed in Tables I and II. See the text for the details.

contribution from all resonances to the total cross section of $\gamma p \to K^+\Sigma^0(1385)$ is important in the region around $E_\gamma = 1.8 \sim 2.0$ GeV. Although the contribution coming from the missing resonances (dash-dash-dotted line) is small compared to the PDG resonance contributions (dotted line), it moves the peak coming from the resonant terms to a somewhat higher energy. This result shows that most resonance contributions come from the sum of the PDG resonances. However, it should be kept in mind that this conclusion follows from the quark model predictions of Refs. [10, 30] for the empirically not well-known decay properties of the PDG resonances. Therefore, detailed studies on this reaction could be used to constrain the properties of the PDG resonances listed in Table I. The total cross sections obtained by including only the PDG resonances and all the resonances considered in the present work, which also includes the missing resonances predicted by quark models, are given, respectively, by the solid line and the dot-dashed line in Fig. 3. These results show that the peak observed in the preliminary CLAS data can be successfully explained by these resonances.

The contributions from different resonances are shown

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[3] The preliminary CLAS data give very small cross sections for $E_\gamma \leq 1.7$ GeV, which deviate significantly from our prediction. These two data points are now corrected in the new analyses of the CLAS data which are in progress [14].
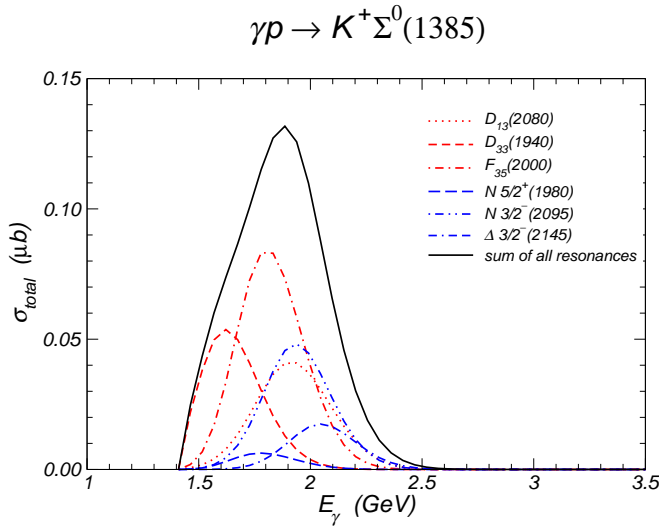
FIG. 4: (Color online) Contributions from various resonances to the total cross section for $\gamma p \to K^+\Sigma^0(1385)$.

FIG. 5: (Color online) Differential cross sections for $\gamma p \to K^+\Sigma^0(1385)$ at $E_\gamma = 1.7$, 1.8, 1.9, and 2.3 GeV with the inclusion of resonances. The dash-dash-dotted line is the sum of the resonance terms. See the text for the details.

FIG. 6: (Color online) Single photon asymmetry for the reaction $\gamma p \to K^+\Sigma^0(1385)$ at $E_\gamma = 1.7$, 1.8, 1.9, and 2.3 GeV. Dashed lines are without the resonance contributions. Solid lines are obtained with the PDG resonances, whereas the dot-dashed lines are with all resonances considered in the present work.

separately in Fig. 4. The solid line is the sum of all the resonances considered in this work and, therefore, corresponds to the dashed line in Fig. 3. The largest contribution comes from the Δ resonance $\Delta(2000)F_{35}$ (the dot-dashed line in Fig. 4), and the contributions from $\Delta(1940)D_{33}$ (the short dashed line in Fig. 4) and $N(2080)D_{13}$ (the dotted line in Fig. 4) are also noticeable. One interesting result is that the contribution from the missing resonance $N\frac{3}{2}^-(2095)$ is not the dominant one, although it is as large as that from $N(2080)D_{13}$. As discussed above, this missing resonance is predicted to have a very large coupling to $K\Sigma(1385)$. Its effect in the reaction $\gamma p \to K^+\Sigma^0(1385)$ is, however, not large as a result of its rather small couplings to $N\gamma$. Furthermore, this resonance has a destructive interference with the other missing resonance, $\Delta\frac{3}{2}^-(2145)$, so that the net contribution from missing resonances becomes small. For $N(2090)S_{11}$ and $N(2200)D_{15}$, their contributions are found to be too small to be shown in Fig. 4.

B. Differential cross section and photon asymmetry

Similar conclusions on the role of resonances in the reaction $\gamma p \to K^+\Sigma^0(1385)$ can be drawn from its differential cross sections shown in Fig. 5. The solid and dashed lines, which are obtained with the PDG resonances and with all resonances, respectively, are close to each other, but they can be distinguished from the dotted lines that are obtained without the resonant contribution, provided the data are accurate enough, in particular, in the region of $E_\gamma = 1.8 \sim 1.9$ GeV. At higher energies, the models with and without resonances give nearly the same result. The contributions from the sum of all the resonances considered in the present work are given by the dot-dashed lines.
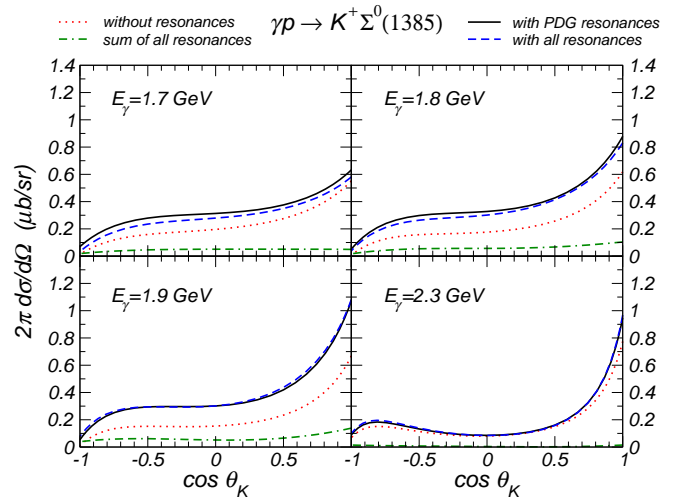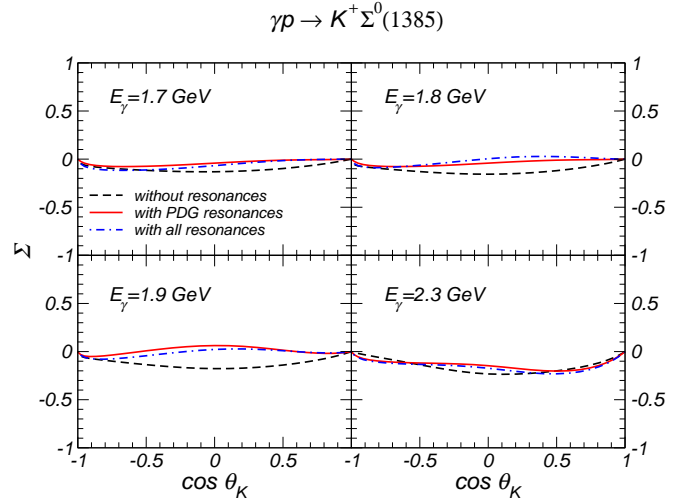
We next consider the photon single asymmetry in the reaction $\gamma p \to K^+\Sigma^0(1385)$, which is defined as

$$\Sigma = \frac{d\sigma_x/d\Omega - d\sigma_y/d\Omega}{d\sigma_x/d\Omega - d\sigma_y/d\Omega}, \qquad (19)$$

where $d\sigma_x/d\Omega$ and $d\sigma_y/d\Omega$ are the differential cross sections with linearly polarized photons in the x-direction and in the y-direction, respectively. Here, the x-direction and the beam momentum direction (i.e., the z-direction) define the reaction plane, and the y-direction is transverse to the reaction plane. The results are shown in

Fig. 6, and it is seen that the role of the resonances can be verified by measuring the photon asymmetry.[4] The similarities observed between the solid line (obtained with PDG resonances) and the dot-dashed line (obtained with all resonances) make it, however, difficult to identify the role of the missing resonances.

IV. SUMMARY AND DISCUSSION

In this paper, we have studied the reaction mechanisms for $K\Sigma(1385)$ production in photon-proton collisions. We find that the peak observed in the preliminary total cross section data of the CLAS Collaboration requires the inclusion of the resonance contribution in the production mechanism. We have accounted for the role of the resonances based on the effective Lagrangian approach. In the present work, we have considered eight nucleon and Δ resonances. Five of them are listed in PDG (Table I) and three of them are missing resonances predicted by the quark model (Table II). However, the properties of these resonances are poorly known or unknown even for the PDG resonances, and we have thus relied on the predictions of hadronic models for the resonance parameters. In particular, we have related the amplitudes of $R \to N\gamma$ and $R \to K\Sigma(1385)$ decays with the coupling constants of our effective Lagrangian, and the predictions of a quark model made in Refs. [10, 30] for the decay amplitudes are then used to determine these coupling constants.

The results obtained in this work show that the most important contribution comes from $\Delta(2000)F_{35}$, and the contributions of $\Delta(1940)D_{33}$ and $N(2080)D_{13}$ are also important. Among the missing resonances, the $N\frac{3}{2}^-(2095)$ contribution is comparable to those of $\Delta(1940)D_{33}$ and $N(2080)D_{13}$. Although this resonance has the largest partial decay width into the $K\Sigma(1385)$ channel, its small photon helicity amplitudes into $N\gamma$ reduces its contribution to this reaction. Furthermore, the contributions from the missing resonances are found to have destructive interference with other missing resonances, and this makes the sum of the missing resonance terms rather small. This is also verified by our results on the photon single asymmetry in this reaction. Our predictions for the cross section and photon single asymmetry show a significant difference between the models with and without the resonances, and this can be verified by experiments at the currently available electron/photon facilities.

It should be stressed that further works to unravel the properties of the resonances is strongly required. For example, some other missing resonances such as $N\frac{1}{2}^-(2070)$ and $N\frac{5}{2}^-(2260)$ are predicted in Ref. [10] to have large

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[4] It should, however, be noted that the final-state interactions may give nontrivial contributions to spin asymmetries.

couplings to the $K\Sigma(1385)$ channel. However, we could not study their role in the reaction $\gamma p \to K^+\Sigma^0(1385)$ as their photon helicity amplitudes are unknown in the same quark model. Furthermore, the properties of resonance decays into $K\Sigma(1385)$ and $N\gamma$ should also be investigated by other models of hadron structure. This would help us to improve our understanding of the resonances and search for the missing ones. Finally, to identify the role of resonances of different isospin, it is desirable to study $K\Sigma(1385)$ photoproduction in other isospin channels.

APPENDIX A: PROPAGATORS AND ISOSPIN STRUCTURE

The propagator of a spin-3/2 Rarita-Schwinger field of momentum p and mass M reads as

$$\Delta_{\alpha\beta}(p, M) = \frac{\not{p} - M}{p^2 - M^2} S_{\alpha\beta}(p, M), \tag{A1}$$

where

$$S_{\alpha\beta}(p, M) = -\bar{g}_{\alpha\beta} + \frac{1}{3}\bar{\gamma}_\alpha\bar{\gamma}_\beta. \tag{A2}$$

With

$$\begin{aligned}
\bar{g}_{\mu\nu} &= g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}, \\
\bar{\gamma}_\mu &= \gamma_\mu - \frac{p_\mu}{M^2}\not{p},
\end{aligned} \tag{A3}$$

this leads to

$$\begin{aligned}
S_{\alpha\beta}(p, M) &= -g_{\alpha\beta} + \frac{1}{3}\gamma_\alpha\gamma_\beta + \frac{1}{3M}\left(\gamma_\alpha p_\beta - p_\alpha\gamma_\beta\right) \\
&\quad + \frac{2}{3M^2}p_\alpha p_\beta.
\end{aligned} \tag{A4}$$

The propagator of a spin-5/2 baryon of momentum p and mass M is written as [32–35]

$$\Delta_{\alpha\beta;\mu\nu}(p, M) = \frac{\not{p} - M}{p^2 - M^2} S_{\alpha\beta;\mu\nu}(p, M), \tag{A5}$$

where

$$S_{\alpha\beta;\mu\nu}(p,M) = \frac{1}{2}\left(\bar{g}_{\alpha\mu}\bar{g}_{\beta\nu} + \bar{g}_{\alpha\nu}\bar{g}_{\beta\mu}\right) - \frac{1}{5}\bar{g}_{\alpha\beta}\bar{g}_{\mu\nu}$$
$$- \frac{1}{10}\left(\bar{\gamma}_{\alpha}\bar{\gamma}_{\mu}\bar{g}_{\beta\nu} + \bar{\gamma}_{\alpha}\bar{\gamma}_{\nu}\bar{g}_{\beta\mu} + \bar{\gamma}_{\beta}\bar{\gamma}_{\mu}\bar{g}_{\alpha\nu} + \bar{\gamma}_{\beta}\bar{\gamma}_{\nu}\bar{g}_{\alpha\mu}\right).$$
$$(A6)$$

For resonances with finite width Γ, the mass M in the propagator is replaced by $M - i\Gamma/2$.

Since we are considering nucleon and Δ resonances, the resonance field R has either isospin-1/2 or isospin-3/2.

By omitting the space-time indices, the isospin structure of $RK\Sigma^*$ interaction reads as

$$\overline{R}\Sigma^* \cdot \boldsymbol{\tau}K, \qquad (A7)$$

for isospin-1/2 resonance R. If the resonance R has isospin-3/2, the effective Lagrangian has the isospin structure as

$$\overline{R}\boldsymbol{T}_{3/2,1/2} \cdot \boldsymbol{\Sigma}^*K, \qquad (A8)$$

where

$$T_{3/2,1/2}^{(+1)} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad T_{3/2,1/2}^{(0)} = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & \sqrt{2} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad T_{3/2,1/2}^{(-1)} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \sqrt{3} \end{pmatrix}. \qquad (A9)$$

In the interaction Lagrangians presented in Appendix B, the isospin structure given above is always understood.

APPENDIX B: COUPLING CONSTANTS AND DECAY AMPLITUDES

The effective Lagrangians for photoexcitation of a resonance from a nucleon can be written as

$$\mathcal{L}_{RN\gamma}(\tfrac{1}{2}^{\pm}) = \frac{ef_1}{2M_N}\bar{N}\Gamma^{(\mp)}\sigma_{\mu\nu}\partial^{\nu}A^{\mu}R + \text{H.c.},$$

$$\mathcal{L}_{RN\gamma}(\tfrac{3}{2}^{\pm}) = -\frac{ief_1}{2M_N}\overline{N}\Gamma_{\nu}^{(\pm)}F^{\mu\nu}R_{\mu} - \frac{ef_2}{(2M_N)^2}\partial_{\nu}\bar{N}\Gamma^{(\pm)}F^{\mu\nu}R_{\mu} + \text{H.c.},$$

$$\mathcal{L}_{RN\gamma}(\tfrac{5}{2}^{\pm}) = \frac{ef_1}{(2M_N)^2}\bar{N}\Gamma_{\nu}^{(\mp)}\partial^{\alpha}F^{\mu\nu}R_{\mu\alpha} - \frac{ief_2}{(2M_N)^3}\partial_{\nu}\bar{N}\Gamma^{(\mp)}\partial^{\alpha}F^{\mu\nu}R_{\mu\alpha} + \text{H.c.}, \qquad (B1)$$

for $j^{\pi} = \frac{1}{2}^{\pm}$, $\frac{3}{2}^{\pm}$, and $\frac{5}{2}^{\pm}$ resonances. In the above, A^{μ} is the photon field with $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$; R, R_{μ}, and $R_{\mu\nu}$ are the spin-1/2, spin-3/2, and spin-5/2 resonance fields, respectively; and $\Gamma_{\mu}^{(\pm)}$ and $\Gamma^{(\pm)}$ are defined in Eq. (11).

The coupling constants f_1 and f_2 in $\mathcal{L}_{RN\gamma}$ are related to the photon helicity amplitudes of the resonance R, which are defined as

$$\Gamma(R \to N\gamma) = \frac{k_{\gamma}^2}{\pi}\frac{2M_N}{(2j+1)M_R}\left[|A_{1/2}|^2 + |A_{3/2}|^2\right], \qquad (B2)$$

where $k_{\gamma} = (M_R^2 - M_N^2)/(2M_R)$ and M_R is the resonance mass. With our effective Lagrangians, the helicity amplitudes are expressed as [36]

$$A_{1/2}(\tfrac{1}{2}^{\pm}) = \mp\frac{ef_1}{2M_N}\sqrt{\frac{k_{\gamma}M_R}{M_N}},$$

$$A_{1/2}(\tfrac{3}{2}^{\pm}) = \mp\frac{e\sqrt{6}}{12}\sqrt{\frac{k_{\gamma}}{M_NM_R}}\left[f_1 + \frac{f_2}{4M_N^2}M_R(M_R \mp M_N)\right],$$

$$A_{3/2}(\tfrac{3}{2}^{\pm}) = \mp\frac{e\sqrt{2}}{4M_N}\sqrt{\frac{k_{\gamma}M_R}{M_N}}\left[f_1 \mp \frac{f_2}{4M_N}(M_R \mp M_N)\right],$$

$$A_{1/2}(\tfrac{5}{2}^{\pm}) = \pm\frac{e}{4\sqrt{10}}\frac{k_{\gamma}}{M_N}\sqrt{\frac{k_{\gamma}}{M_NM_R}}\left[f_1 + \frac{f_2}{4M_N^2}M_R(M_R \pm M_N)\right],$$

$$A_{3/2}(\tfrac{5}{2}^{\pm}) = \pm\frac{e}{4\sqrt{5}}\frac{k_{\gamma}}{M_N^2}\sqrt{\frac{k_{\gamma}M_R}{M_N}}\left[f_1 \pm \frac{f_2}{4M_N}(M_R \pm M_N)\right], \qquad (B3)$$

where the spin-parity of the resonance is given in parentheses.

For the interaction Lagrangians describing the decay of a baryon resonance R of spin-parity j^π to $K\Sigma(1385)$ of the spin-parity combination $0^- + \frac{3}{2}^+$, consideration of angular momentum and parity conservation leads to only one term for $j^\pi = \frac{1}{2}^\pm$ resonances and two terms for resonances with $j \geq \frac{3}{2}$. The general form of these interaction Lagrangians can be written as

$$
\begin{aligned}
\mathcal{L}_{RK\Sigma^*}(\tfrac{1}{2}^\pm) &= \frac{h_1}{M_K} \partial_\mu K \bar{\Sigma}^{*\mu} \Gamma^{(\mp)} R + \text{H.c.}, \\
\mathcal{L}_{RK\Sigma^*}(\tfrac{3}{2}^\pm) &= \frac{h_1}{M_K} \partial^\alpha K \bar{\Sigma}^{*\mu} \Gamma_\alpha^{(\pm)} R_\mu + \frac{ih_2}{M_K^2} \partial^\mu \partial^\alpha K \bar{\Sigma}_\alpha^* \Gamma^{(\pm)} R_\mu + \text{H.c.}, \\
\mathcal{L}_{RK\Sigma^*}(\tfrac{5}{2}^\pm) &= \frac{ih_1}{M_K^2} \partial^\mu \partial^\beta K \bar{\Sigma}^{*\alpha} \Gamma_\mu^{(\mp)} R_{\alpha\beta} - \frac{h_2}{M_K^3} \partial^\mu \partial^\alpha \partial^\beta K \bar{\Sigma}_\mu^* \Gamma^{(\mp)} R_{\alpha\beta} + \text{H.c.}.
\end{aligned}
\tag{B4}
$$

The decay width of resonance R into $K\Sigma(1385)$ is then obtained as

$$
\begin{aligned}
\Gamma(\tfrac{1}{2}^\pm \to K\Sigma^*) &= \frac{h_1^2}{2\pi} \frac{q^3 M_R}{M_K^2 M_{\Sigma^*}^2} (E_{\Sigma^*} \pm M_{\Sigma^*}), \\
\Gamma(\tfrac{3}{2}^\pm \to K\Sigma^*) &= \frac{1}{24\pi} \frac{q}{M_R M_{\Sigma^*}^2} (E_{\Sigma^*} \mp M_{\Sigma^*}) \Bigg\{ \frac{h_1^2}{M_K^2} (M_R \pm M_{\Sigma^*})^2 \left(2E_{\Sigma^*}^2 \mp 2E_{\Sigma^*} M_{\Sigma^*} + 5M_{\Sigma^*}^2 \right) \\
&\qquad \mp 2\frac{h_1 h_2}{M_K^3} M_R q^2 (M_R \pm M_{\Sigma^*})(2E_{\Sigma^*} \mp M_{\Sigma^*}) + 2\frac{h_2^2}{M_K^4} M_R^2 q^4 \Bigg\}, \\
\Gamma(\tfrac{5}{2}^\pm \to K\Sigma^*) &= \frac{1}{60\pi} \frac{q^3}{M_R M_{\Sigma^*}^2} (E_{\Sigma^*} \pm M_{\Sigma^*}) \Bigg\{ \frac{h_1^2}{M_K^4} (M_R \mp M_{\Sigma^*})^2 \left(4E_{\Sigma^*}^2 \pm 4E_{\Sigma^*} M_{\Sigma^*} + 7M_{\Sigma^*}^2 \right) \\
&\qquad \mp 4\frac{h_1 h_2}{M_K^5} M_R q^2 (M_R \mp M_{\Sigma^*})(2E_{\Sigma^*} \pm M_{\Sigma^*}) + 4\frac{h_2^2}{M_K^6} M_R^2 q^4 \Bigg\},
\end{aligned}
\tag{B5}
$$

depending on its spin-parity. In the above, q is the magnitude of the three-momenta of final-state particles in the rest frame of the resonance,

$$
q = \frac{1}{2M_R} \sqrt{[M_R^2 - (M_{\Sigma^*} + M_K)^2][M_R^2 - (M_{\Sigma^*} - M_K)^2]},
\tag{B6}
$$

and $E_{\Sigma^*} = \sqrt{M_{\Sigma^*}^2 + q^2}$. Above formulas are valid for the decays of resonances of isospin-1/2 as well as isospin-3/2.

For a $j = \frac{1}{2}$ resonance, the decay width can be used to determine the magnitude of the coupling constant h_1 but not its phase. For resonances with $j \geq \frac{3}{2}$, this gives only one relation for two coupling constants, h_1 and h_2. Therefore, we need to know the decay amplitudes to uniquely determine the coupling constants. The signs of the couplings are then fixed by hadron model predictions.

The decay amplitude for $R \to K\Sigma(1385)$ can be written as

$$
\langle K(\boldsymbol{q})\, \Sigma^*(-\boldsymbol{q}, m_f)| - i\, \mathcal{H}_{\text{int}}|R(\mathbf{0}, m_j)\rangle = 2\pi M_R \sqrt{\frac{2}{q}} \sum_{\ell, m_\ell} \langle \ell\, m_\ell\, \tfrac{3}{2}\, m_f | j\, m_j \rangle Y_{\ell m_\ell}(\hat{\boldsymbol{q}}) G(\ell),
\tag{B7}
$$

where $Y_{\ell m_\ell}(\hat{\boldsymbol{q}})$ and $\langle \ell\, m_\ell\, \tfrac{3}{2}\, m_f | j\, m_j \rangle$ are the spherical harmonics and Clebsch-Gordan coefficient, respectively. This also defines the partial wave decay amplitude $G(\ell)$. The relative orbital angular momentum ℓ of the final state is constrained by the spin-parity of the resonance. The decay width is then given by

$$
\Gamma(R \to K\Sigma^*) = \sum_\ell |G(\ell)|^2,
\tag{B8}
$$

where the values of $G(\ell)$ can be obtained from the prediction of hadronic models, for example, in Ref. [10].

For the decay of a $j^\pi = \frac{1}{2}^\pm$ resonance, angular momentum conservation restricts the relative orbital angular momentum to $\ell = 1, 2$. For the decay of a positive parity resonance, the final-state particles are therefore in the relative p wave, while they are in the relative d-wave in the decay of a negative parity resonance. In this case, we have

$$
G(1) = -\frac{1}{\sqrt{2\pi}} \frac{q}{M_{\Sigma^*}} \sqrt{qM_R} \sqrt{E_{\Sigma^*} + M_{\Sigma^*}} \frac{h_1}{M_K},
\tag{B9}
$$

for a $j^\pi = \frac{1}{2}^+$ resonance, and

$$G(2) = -\frac{1}{\sqrt{2\pi}} \frac{q}{M_{\Sigma^*}} \sqrt{qM_R} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}} \frac{h_1}{M_K}, \tag{B10}$$

for a $j^\pi = \frac{1}{2}^-$ resonance.

For a resonance of $j = \frac{3}{2}$, the final $K\Sigma(1385)$ state is in the relative p and f waves in the decay of a positive parity resonance and are in the relative s and d waves in the decay of a negative parity resonance. The decay amplitudes can be written in terms of the coupling constants as

$$\begin{aligned} G(1) &= G_{11}^{(3/2)} \frac{h_1}{M_K} + G_{12}^{(3/2)} \frac{h_2}{M_K^2}, \\ G(3) &= G_{31}^{(3/2)} \frac{h_1}{M_K} + G_{32}^{(3/2)} \frac{h_2}{M_K^2}, \end{aligned} \tag{B11}$$

for a positive parity resonance, where

$$\begin{aligned} G_{11}^{(3/2)} &= \frac{\sqrt{30}}{60\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \sqrt{\frac{q}{M_R}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}} (M_R + M_{\Sigma^*})(E_{\Sigma^*} + 4M_{\Sigma^*}), \\ G_{12}^{(3/2)} &= -\frac{\sqrt{30}}{60\sqrt{\pi}} \frac{q^2 \sqrt{qM_R}}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}} \\ G_{31}^{(3/2)} &= -\frac{\sqrt{30}}{20\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \sqrt{\frac{q}{M_R}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}} (M_R + M_{\Sigma^*})(E_{\Sigma^*} - M_{\Sigma^*}), \\ G_{32}^{(3/2)} &= \frac{\sqrt{30}}{20\sqrt{\pi}} \frac{q^2 \sqrt{qM_R}}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*} - M_{\Sigma^*}}, \end{aligned} \tag{B12}$$

and

$$\begin{aligned} G(0) &= G_{01}^{(3/2)} \frac{h_1}{M_K} + G_{02}^{(3/2)} \frac{h_2}{M_K^2}, \\ G(2) &= G_{21}^{(3/2)} \frac{h_1}{M_K} + G_{22}^{(3/2)} \frac{h_2}{M_K^2}, \end{aligned} \tag{B13}$$

for a negative parity resonance, where

$$\begin{aligned} G_{01}^{(3/2)} &= \frac{\sqrt{6}}{12\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \sqrt{\frac{q}{M_R}} \sqrt{E_{\Sigma^*} + M_{\Sigma^*}} (M_R - M_{\Sigma^*})(E_{\Sigma^*} + 2M_{\Sigma^*}), \\ G_{02}^{(3/2)} &= \frac{\sqrt{6}}{12\sqrt{\pi}} \frac{q^2 \sqrt{qM_R}}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*} + M_{\Sigma^*}} \\ G_{21}^{(3/2)} &= -\frac{\sqrt{6}}{12\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \sqrt{\frac{q}{M_R}} \sqrt{E_{\Sigma^*} + M_{\Sigma^*}} (M_R - M_{\Sigma^*})(E_{\Sigma^*} - M_{\Sigma^*}), \\ G_{22}^{(3/2)} &= -\frac{\sqrt{6}}{12\sqrt{\pi}} \frac{q^2 \sqrt{qM_R}}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*} + M_{\Sigma^*}}. \end{aligned} \tag{B14}$$

In the decay of a spin-5/2 resonance into $K\Sigma(1385)$, the final state is in the relative p and f waves for a positive parity resonance and is in the relative d and g waves for a negative parity resonance. The decay amplitudes are then written as

$$\begin{aligned} G(1) &= G_{11}^{(5/2)} \frac{h_1}{M_K^2} + G_{12}^{(5/2)} \frac{h_2}{M_K^3}, \\ G(3) &= G_{31}^{(5/2)} \frac{h_1}{M_K^2} + G_{32}^{(5/2)} \frac{h_2}{M_K^3}, \end{aligned} \tag{B15}$$

for a positive parity resonance, where

$$G_{11}^{(5/2)} = -\frac{1}{10\sqrt{\pi}}\frac{q}{M_{\Sigma^*}}\sqrt{\frac{q}{M_R}}\sqrt{E_{\Sigma^*}+M_{\Sigma^*}}(M_R-M_{\Sigma^*})(2E_{\Sigma^*}+3M_{\Sigma^*}),$$

$$G_{12}^{(5/2)} = \frac{1}{5\sqrt{\pi}}\frac{q^3\sqrt{qM_R}}{M_{\Sigma^*}}\sqrt{E_{\Sigma^*}+M_{\Sigma^*}},$$

$$G_{31}^{(5/2)} = -\frac{\sqrt{6}}{15\sqrt{\pi}}\frac{q}{M_{\Sigma^*}}\sqrt{\frac{q}{M_R}}\sqrt{E_{\Sigma^*}+M_{\Sigma^*}}(M_R-M_{\Sigma^*})(E_{\Sigma^*}-M_{\Sigma^*}),$$

$$G_{32}^{(5/2)} = -\frac{\sqrt{6}}{15\sqrt{\pi}}\frac{q^3\sqrt{qM_R}}{M_{\Sigma^*}}\sqrt{E_{\Sigma^*}+M_{\Sigma^*}}, \tag{B16}$$

and

$$G(2) = G_{21}^{(5/2)}\frac{h_1}{M_K^2} + G_{22}^{(5/2)}\frac{h_2}{M_K^3},$$

$$G(4) = G_{41}^{(5/2)}\frac{h_1}{M_K^2} + G_{42}^{(5/2)}\frac{h_2}{M_K^3}, \tag{B17}$$

for a negative parity resonance, where

$$G_{21}^{(5/2)} = -\frac{\sqrt{105}}{210\sqrt{\pi}}\frac{q}{M_{\Sigma^*}}\sqrt{\frac{q}{M_R}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}}(M_R+M_{\Sigma^*})(2E_{\Sigma^*}+5M_{\Sigma^*}),$$

$$G_{22}^{(5/2)} = -\frac{\sqrt{105}}{105\sqrt{\pi}}\frac{q^3\sqrt{qM_R}}{M_{\Sigma^*}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}},$$

$$G_{41}^{(5/2)} = \frac{\sqrt{70}}{35\sqrt{\pi}}\frac{q}{M_{\Sigma^*}}\sqrt{\frac{q}{M_R}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}}(M_R+M_{\Sigma^*})(E_{\Sigma^*}-M_{\Sigma^*}),$$

$$G_{42}^{(5/2)} = \frac{\sqrt{70}}{35\sqrt{\pi}}\frac{q^3\sqrt{qM_R}}{M_{\Sigma^*}}\sqrt{E_{\Sigma^*}-M_{\Sigma^*}}. \tag{B18}$$

It can be verified that the decay widths obtained from Eq. (B8) with the above relations reproduce the results given in Eq. (B5). Above effective Lagrangians are constructed for resonances of spin up to 5/2, but they can be straightforwardly generalized to resonances with arbitrary spin-parity.

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